MODELLING OF DYNAMICAL SYSTEMS

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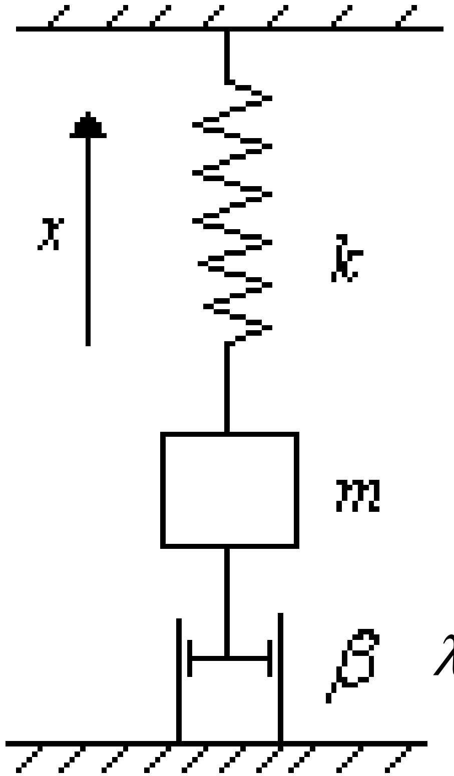
OUTLINE

1. **INTRODUCTION**
2. **MODELLING OF THE “MASS-SPRING-DAMPER” MECHANICAL SYSTEM**
3. **MODELLING OF THE “MASS-SPRING” MECHANICAL SYSTEM CONSTRAINED TO SINUSOIDAL INPUT SIGNAL**
4. **MATHEMATICAL MODELS OF THE STOCHASTIC CONTINUOUS ATMOSPHERIC DISTURBANCES APPLIED FOR IDENTIFICATION PURPOSES**
5. **CONCLUSIONS**

INTRODUCTION

1. Importance of modeling
   - Analysis
   - Identification
   - Preliminary design
2. Modeling physical systems
   - Mechanics
   - Electrical Systems
   - Magnetic systems
   - Fluid dynamics
3. Modeling deterministic, and stochastic systems, and signals.

MODELLING OF THE "MASS-SPRING-DAMPER" MECHANICAL SYSTEM



$$\frac{d(m\mathrm{v})}{dt} = \Sigma F_i \qquad \Sigma F_i = kx + \beta \frac{dx}{dt}$$

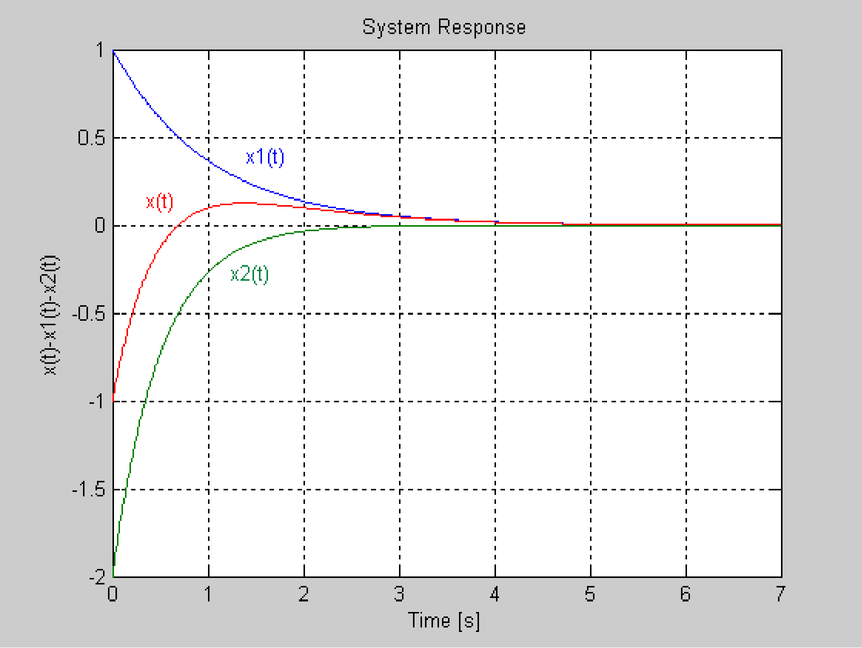
$$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = 0 \qquad x(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

$$m\lambda^2 + \beta\lambda + k = 0$$

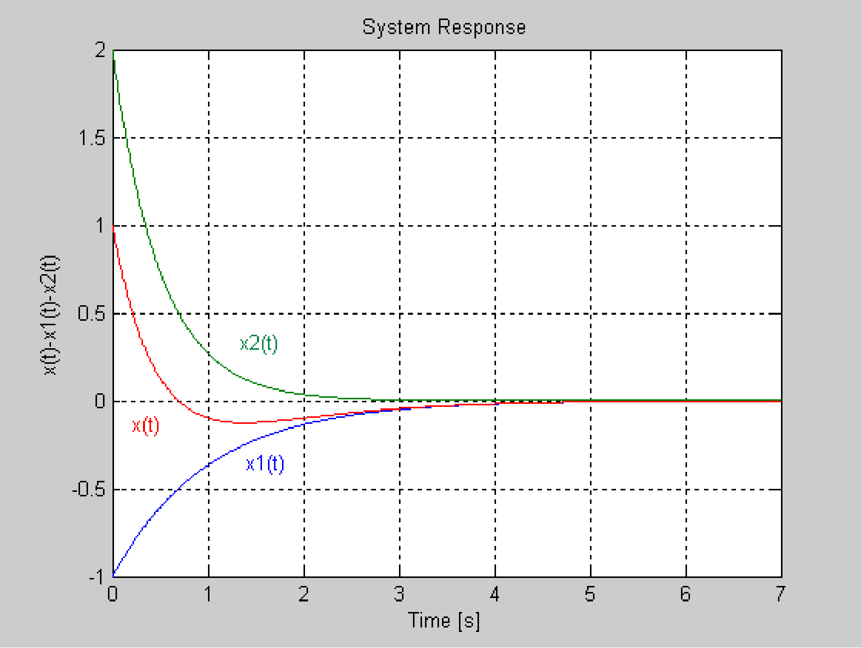
$$\lambda_1 = -\frac{\beta}{2m} + \sqrt{\left(\frac{\beta}{2m}\right)^2 - \frac{k}{m}}; \qquad \lambda_2 = -\frac{\beta}{2m} - \sqrt{\left(\frac{\beta}{2m}\right)^2 - \frac{k}{m}}$$

MODELLING OF THE "MASS-SPRING-DAMPER" MECHANICAL SYSTEM – CONT'D

$$\beta^2 > 4 \cdot km$$



$$A = 1;\ B = -2;\ \lambda_1 = -1;\ \lambda_2 = -2$$



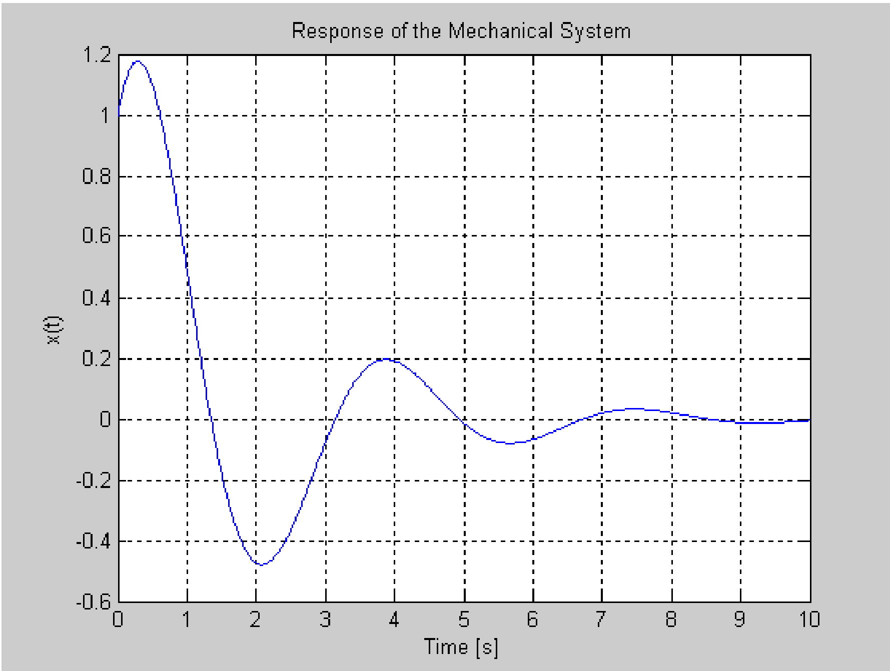
$$A = -1;\ B = 2;\ \lambda_1 = -1;\ \lambda_2 = -2$$

MODELLING OF THE “MASS-SPRING-DAMPER” MECHANICAL SYSTEM – CONT’D

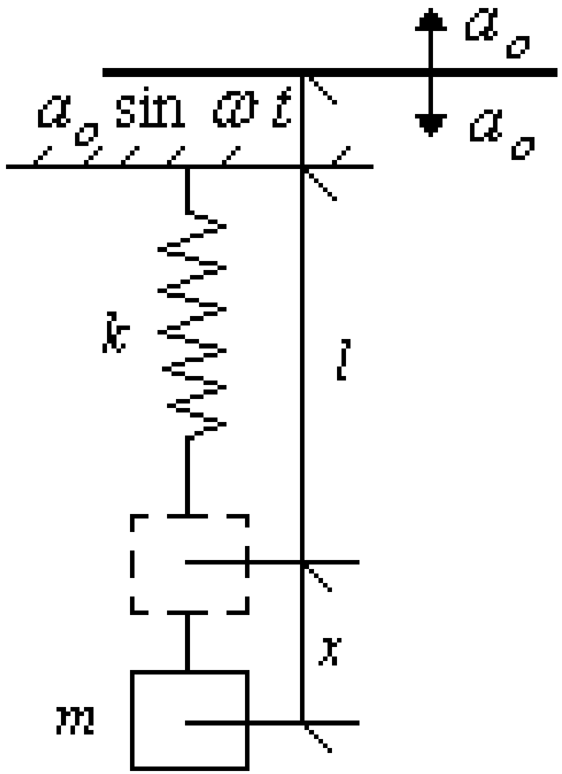
$$\beta^2 < 4 \cdot k \cdot m$$

$$x(t) = C\,e^{-\frac{\beta}{2m}t}\cos\left(\sqrt{\frac{k}{m}-\left(\frac{\beta}{2m}\right)^2}\;t\right) + D\,e^{-\frac{\beta}{2m}t}\sin\left(\sqrt{\frac{k}{m}-\left(\frac{\beta}{2m}\right)^2}\;t\right) \qquad \frac{\beta}{m}=1;\quad \frac{k}{m}=2,\quad m=1\,kg$$

$$x(t) = e^{-0{,}5\;t}\cos\left(1{,}75\,t\right) + e^{-0{,}5\;t}\sin\left(1{,}75\,t\right)$$



MODELLING OF THE "MASS-SPRING" MECHANICAL SYSTEM CONSTRAINED TO SINUSOIDAL INPUT SIGNAL



$$m\frac{d^2x}{dt^2} + kx = a_o \sin \omega t$$

$$a_o \sin \omega t + l + x$$

$$m\frac{d^2}{dt^2}(a_o \sin \omega t + l + x) + kx = 0$$

$$m\frac{d^2x}{dt^2} + kx = m a_o \omega^2 \sin \omega t = F_o \sin \omega t$$

MODELLING OF THE "MASS-SPRING" MECHANICAL SYSTEM CONSTRAINED TO SINUSOIDAL INPUT SIGNAL

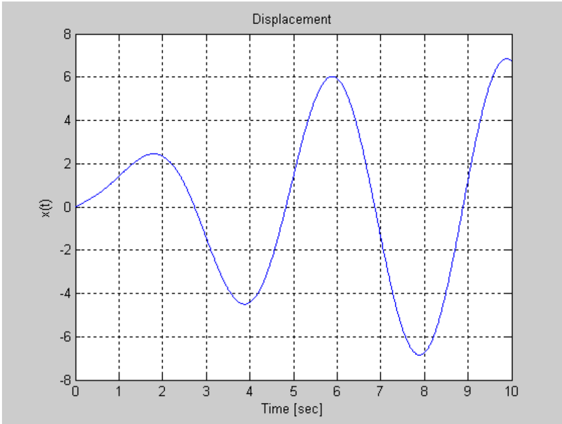
$$x = C \sin \omega t \qquad -mC\omega^2 + kC = F_o \qquad C = \frac{F_o}{k - m\omega^2}$$

$$\omega_o = \sqrt{\frac{k}{m}} \qquad x = \frac{F_o}{m} \frac{\sin \omega t}{\omega_o^2 - \omega^2} = \frac{F_o}{k} \frac{\sin \omega t}{1 - \frac{\omega^2}{\omega_o^2}}$$

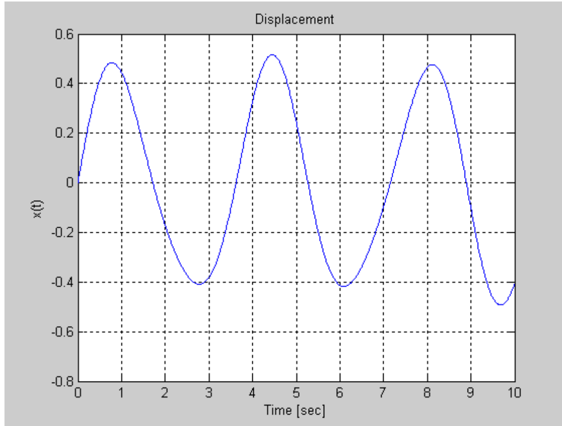
$$x = A \cos \omega_o t + B \sin \omega_o t + \frac{F_o}{m} \frac{\sin \omega t}{\omega_o^2 - \omega^2} \qquad x(0) = 0; \quad \left.\frac{dx}{dt}\right|_{t=0} = \mathrm{v}_o = 1$$

$$x = \mathrm{v}_o \frac{\sin \omega_o t}{\omega_o} + \frac{F_o}{m} \frac{\sin \omega t}{(\omega_o^2 - \omega^2)\omega_o} \left(\omega_o \sin \omega t - \omega \sin \omega t\right)$$

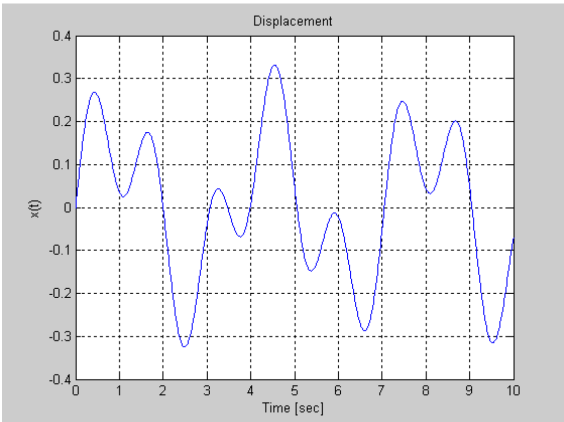
MODELLING OF THE "MASS-SPRING" MECHANICAL SYSTEM CONSTRAINED TO SINUSOIDAL INPUT SIGNAL



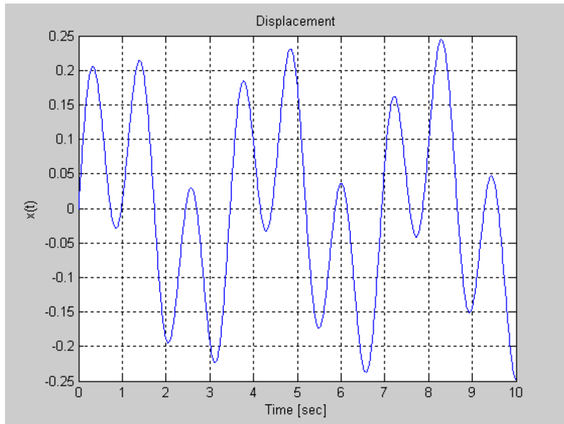
$k = 2$



$k = 10$

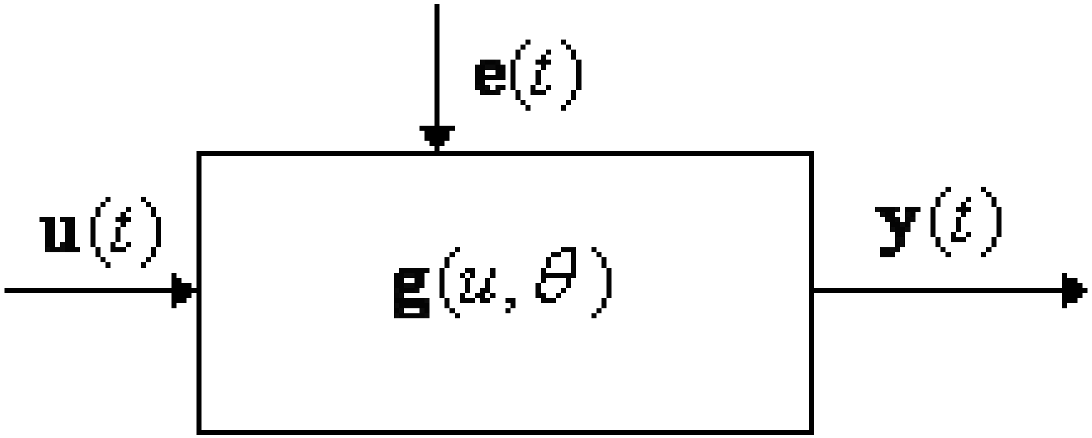


$k = 20$



$k = 30$

MATHEMATICAL MODELS OF THE STOCHASTIC CONTINUOUS ATMOSPHERIC DISTURBANCES APPLIED FOR IDENTIFICATION PURPOSES



$$\mathbf{y}(t) = \mathbf{g}(u, \theta) + \mathbf{e}(t)$$

$$\mathbf{y}(t) = \mathbf{G}\mathbf{u}(t) + \mathbf{H}\mathbf{e}(t)$$

MATHEMATICAL MODELS OF THE STOCHASTIC CONTINUOUS ATMOSPHERIC DISTURBANCES II.

$$\Phi_{Kármán}(\Omega) = \frac{\sigma^2 L}{\pi} \frac{1 + \frac{8}{3}(1{,}339 L\Omega)^2}{\left(1 + 1{,}339 L^2\Omega^2\right)^{11/6}}$$

$$\Phi_{\mathrm{Dryden}}(\Omega) = \frac{\sigma^2 L}{\pi} \frac{1 + 3L^2\Omega^2}{\left(1 + L^2\Omega^2\right)^2}$$

$$\Phi_{\mathrm{u_g}}(\Omega) = \frac{2\sigma_{\mathrm{u}}^2 L_{\mathrm{u}}}{\pi} \frac{1}{1 + (L_{\mathrm{u}}\Omega)^2}$$

$$\Phi_{\mathrm{u_g}}(\omega) = \frac{2\sigma_{\mathrm{u}}^2 L_{\mathrm{u}}}{U_o \pi} \frac{1}{\left\{1 + (L_{\mathrm{u}}/U_o)^2 \omega^2\right\}}$$

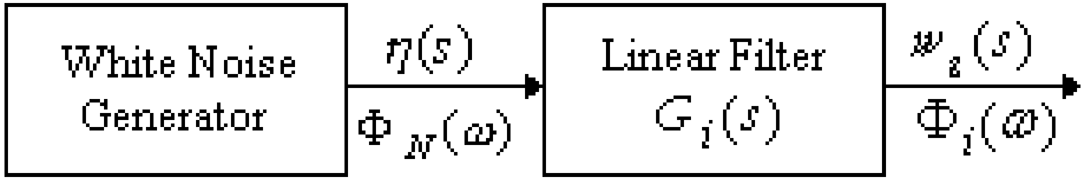
$$\Phi_{\mathrm{v_g}}(\Omega) = \frac{\sigma_{\mathrm{v}}^2 L_{\mathrm{v}}}{\pi} \frac{(1 + 3(L_{\mathrm{v}}\Omega)^2)}{\left[1 + (L_{\mathrm{v}}\Omega)^2\right]^2}$$

$$\Phi_{\mathrm{v_g}}(\omega) = \frac{\sigma_{\mathrm{v}}^2 L_{\mathrm{v}}}{U_o \pi} \frac{(1 + 3\,(L_{\mathrm{v}}/U_o)^2 \omega^2)}{\left\{(1 + (L_{\mathrm{v}}/U_o)^2 \omega^2\right\}^2}$$

$$\Phi_{\mathrm{w_g}}(\Omega) = \frac{\sigma_{\mathrm{w}}^2 L_{\mathrm{w}}}{\pi} \frac{(1 + 3(L_{\mathrm{w}}\Omega)^2)}{\left[1 + (L_{\mathrm{w}}\Omega)^2\right]^2}$$

$$\Phi_{\mathrm{w_g}}(\omega) = \frac{\sigma_{\mathrm{w}}^2 L_{\mathrm{w}}}{U_o \pi} \frac{(1 + 3\,(L_{\mathrm{w}}/U_o)^2 \omega^2)}{\left\{(1 + (L_{\mathrm{w}}/U_o)^2 \omega^2\right\}^2}$$

MATHEMATICAL MODELS OF THE STOCHASTIC CONTINUOUS ATMOSPHERIC DISTURBANCES III.



$$\Phi_i(\omega) = \left| G_i(s) \right|^2_{s=j\omega} \Phi_N(\omega) = G_i(s)\, G_i(-s) \Big|_{s=j\omega} \Phi_N(\omega)$$

$$\Phi_N(\omega) = 1$$

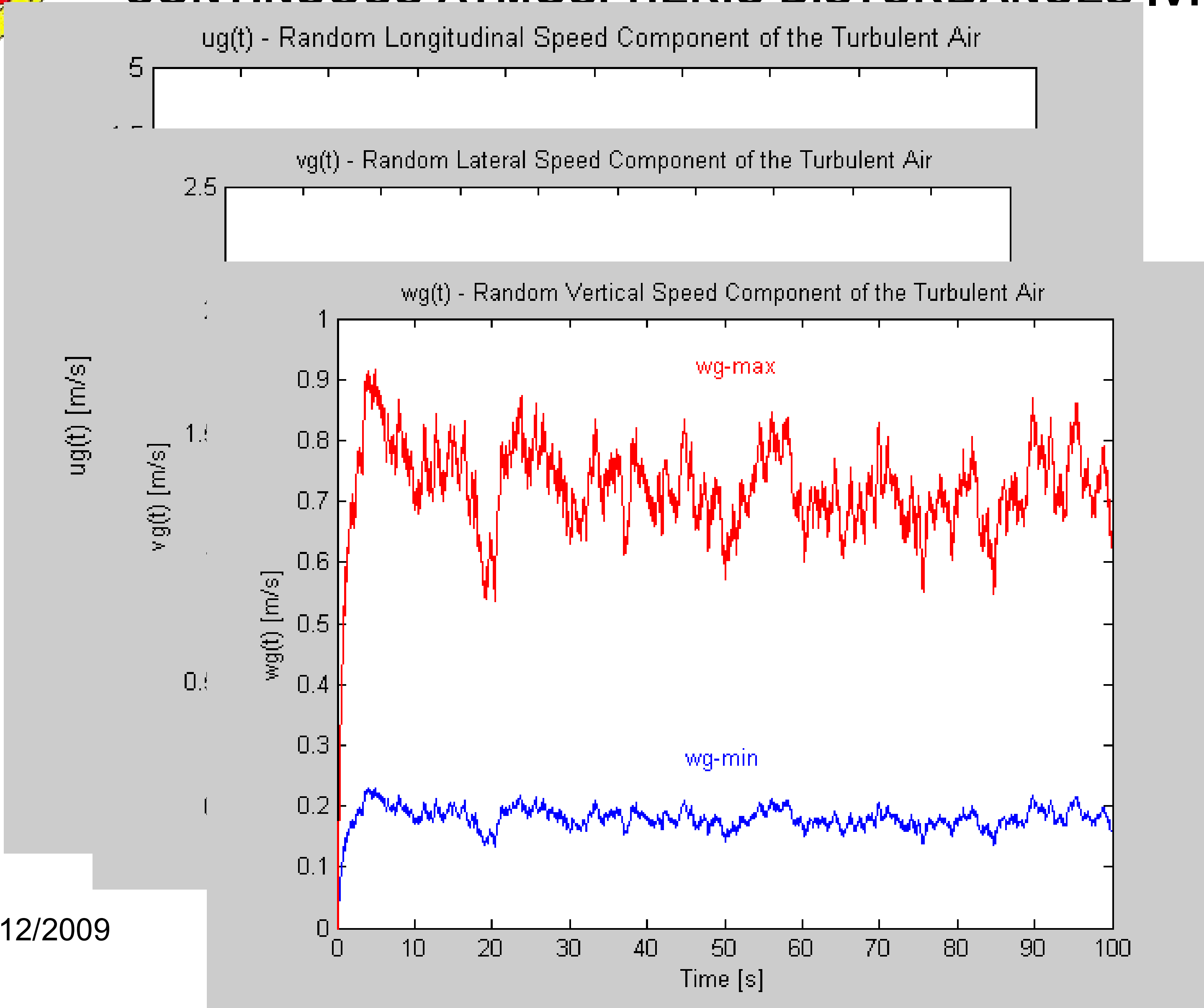
$$G_{u_g}(s) = \frac{\sqrt{K_u}}{s + \lambda_u} \qquad G_{v_g}(s) = \sqrt{K_v}\, \frac{s + \beta_v}{(s + \lambda_v)^2} \qquad G_{w_g}(s) = \sqrt{K_w}\, \frac{s + \beta_w}{(s + \lambda_w)^2}$$

$$K_u = \frac{2U_o \sigma_u^2}{L_u \pi} \qquad K_v = \frac{3U_o \sigma_v^2}{L_v \pi} \qquad K_w = \frac{3U_o \sigma_w^2}{L_w \pi}$$

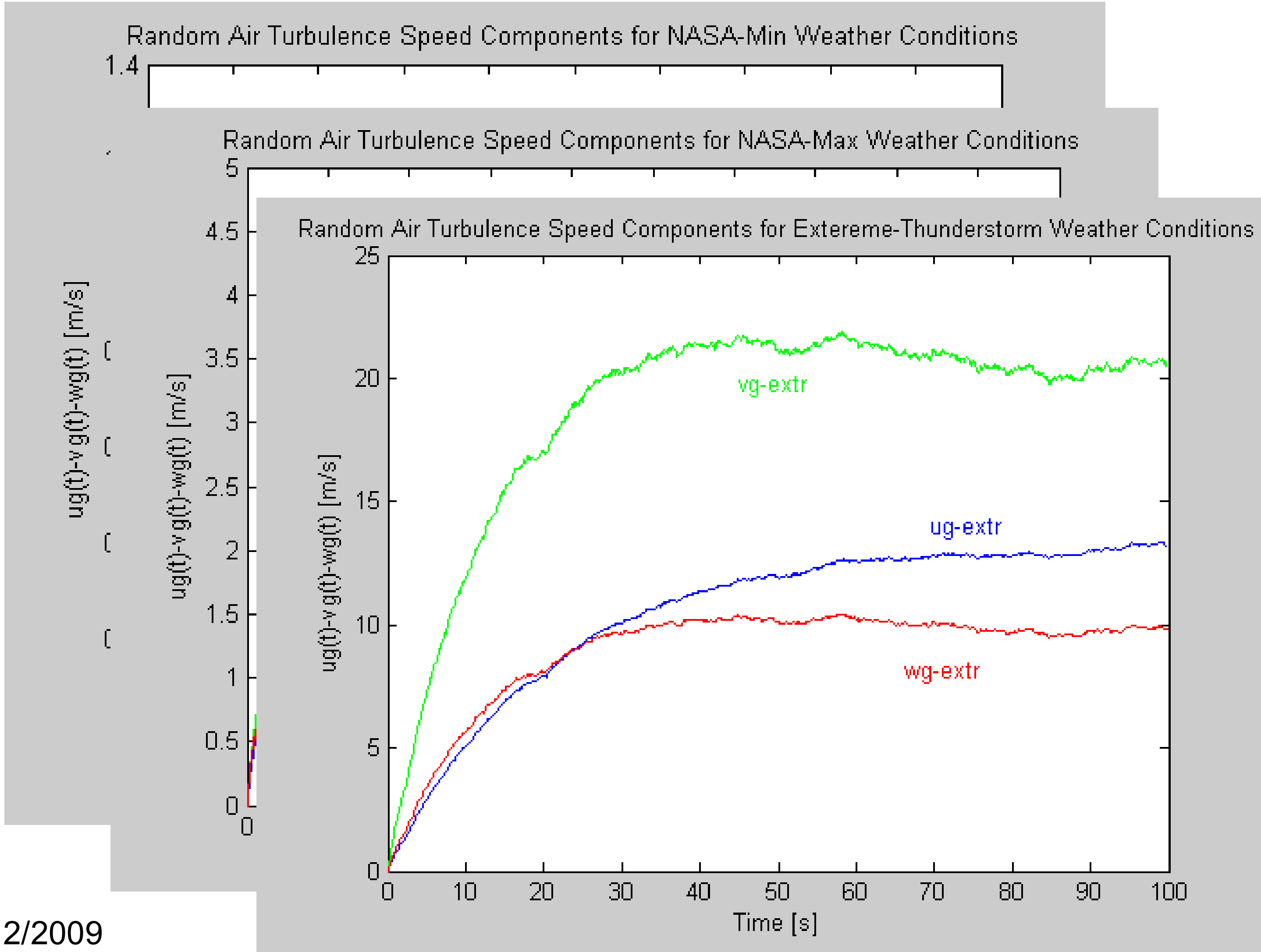
$$\beta_v = \frac{U_o}{\sqrt{3} L_v} \qquad \beta_w = \frac{U_o}{\sqrt{3} L_w}$$

$$\lambda_u = \frac{U_o}{L_u} \qquad \lambda_v = \frac{U_o}{L_v} \qquad \lambda_w = \frac{U_o}{L_w}$$

MATHEMATICAL MODELS OF THE STOCHASTIC CONTINUOUS ATMOSPHERIC DISTURBANCES IV.



MATHEMATICAL MODELS OF THE STOCHASTIC CONTINUOUS ATMOSPHERIC DISTURBANCES V.



CONCLUSIONS

- MODELING GIVES COMPLEX DATA SET OF DYNAMICAL PERFORMANCES
- MAKES
  - POSSIBLE MODEL, AND PARAMETRIC IDENTIFICATION
  - PRELIMINARY DESIGN OF THE CONTROL SYSTEMS
  - SCHEDULING SYSTEM PARAMETERS
  - POSSIBLE TO FIND CLOSED LOOP DYNAMIC PERFORMANCES WHETHER THEY FIT PRELIMINARY DEFINED ONES, OR NOT.

THANKS FOR KIND ATTENTION!

QUESTIONS?